Toy example: projectives and $\mathrm{Ext}^\bullet$: 2×2 upper triangular matrices.

Let A be the algebra of 2×2 upper triangular matrices over $\mathbb{C}$. Write A-mod for the category of left A-modules. We define some distinguished objects in this category:

set $V_1 = \mathbb{C}\text{-span}\{v_1\}$ with an A-action given by $\begin{pmatrix} a & b \\ 0 & c \end{pmatrix} v_1 = c v_1$. set $V_2 = \mathbb{C}\text{-span}\{v_2\}$ with A-module structure given by $\begin{pmatrix} a & b \\ 0 & c \end{pmatrix} v_2 = a v_2$. It is clear that both V_1 and V_2 are simple. In fact, these are all the simple modules (upto isomorphism) in A-mod. ~~Fin~~

Finally, set $P_1 = \mathbb{C}\text{-span}\{v_1, v_2\}$ with $\begin{pmatrix} a & b \\ 0 & c \end{pmatrix} v_1 = c v_1 + b v_2$ and $\begin{pmatrix} a & b \\ 0 & c \end{pmatrix} v_2 = a v_2$. The algebra A itself is an object of A-mod (via multiplication on the left) and $A \simeq P_1 \oplus V_2$ (in A-mod). Thus, both P_1 and V_2 are projective modules. Further, we have a non-split exact sequence:

$$0 \longrightarrow V_2 \longrightarrow P_1 \longrightarrow V_1 \longrightarrow 0.$$

This is also a projective resolution of V_1. Let $X \in A$-mod, and apply the functor $\mathrm{Hom}_A(-, X)$ to this resolution to get the complex of vector spaces:

$$0 \longrightarrow \mathrm{Hom}_A(V_1, X) \longrightarrow \mathrm{Hom}_A(P_1, X) \longrightarrow \mathrm{Hom}_A(V_2, X) \longrightarrow 0.$$

The cohomology of the truncated complex:

$$0 \longrightarrow \mathrm{Hom}_A(P_1, X) \longrightarrow \mathrm{Hom}_A(V_2, X) \longrightarrow 0$$

is by definition $R^\bullet\mathrm{Hom}$. As $\mathrm{Hom}_A(V_1, X)$ is left exact, $R^0\mathrm{Hom}(V_1, X) = \mathrm{Hom}_A(V_1, X)$.

set $X = V_1$ to get

$$0 \longrightarrow \underset{\mathbb{C}}{\underset{\parallel}{\mathrm{Hom}_A(P_1, V_1)}} \longrightarrow \underset{0}{\underset{\parallel}{\mathrm{Hom}_A(V_2, V_1)}} \longrightarrow 0$$

so $R^0\mathrm{Hom}(V_1, V_1) \simeq \mathbb{C}$ and $R^i\mathrm{Hom}(V_1, V_1) = 0$ for $i \neq 0$.

set $X = V_2$ to get

$$0 \longrightarrow \underset{0}{\underset{\parallel}{\mathrm{Hom}_A(P_1, V_2)}} \longrightarrow \underset{\mathbb{C}}{\underset{\parallel}{\mathrm{Hom}_A(V_2, V_2)}} \longrightarrow 0$$

so $R^i\mathrm{Hom}(V_1, V_2) = 0$ for $i \neq 1$ and $R^1\mathrm{Hom}(V_1, V_2) \simeq \mathbb{C}$.