Introduction

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13 Aug 2012

<u>Goal</u> Develop basic tools of representation theory and harmonic analysis in a categorical setting:

$G \curvearrowright$ categories

<u>Why?</u>
- geometry $\to$ $G \curvearrowright X$ ← space
  - ⇝ 'linearize' ⇝ $G \curvearrowright \{ Vect(X), Sh(X), \ldots \}$
- algebra
  - $G \curvearrowright$ algebra $(U(\mathfrak{g}))$ ⇝ $G \curvearrowright$ modules
- physics
  - gauge theories, quantum field theories w/ G-symmetry

Given a category you care about, use symmetry to understand it:

- Lusztig's character sheaves look like characters of groups acting on varieties
  - ⇝ ~~sheav~~ 'sheaves' on G (equivariant under conjugation)
  - ⇝ construct characters of fin. simple groups of Lie type geometrically
- Geometric Langlands
  - categorical analogue of classical Langlands
  - ⇝ analogue of harmonic analysis on arithmetic locally symmetric spaces

$L^2(SL_2\mathbb{Z} \backslash \underbrace{SL_2\mathbb{R} / SO_2}_{\mathbb{H}}) \rightsquigarrow \mathcal{D}$-modules on Bun_G

- Extended topological field theories
  - most field theories in >2 dim, and all field theories in >3 dim are gauge theories $\rightsquigarrow$ involve principal G-bundles

Gauge theory as TFT $\rightsquigarrow$ $G \mapsto$ vector spaces (2d)

$G \mapsto$ categories (3d)

Geometric Langlands $\subset$ $G \mapsto$ 2-categories (4d)

Harmonic analysis

$G \mapsto L^2(X)$
- spectral theory
- fourier analysis
- integral transforms

$G \mapsto Sh(X)$
- need a setting for functional analysis on categories $\rightsquigarrow$ fulfilled by ∞-categories

$\rightsquigarrow$ homological algebra is a basic tool
- studying operations (functors) in a linear setting (abelian categories)

$\rightsquigarrow$ need non-linear version (rings, spaces, ..)

$\rightsquigarrow$ homotopical algebra