~~categorical~~

Fusion categories and module categories

(E. Jenkins

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k - alg. closed field char 0.

finite semisimple categories

All categories are k-linear.

finite = finitely many iso. classes of simples

The tensor product of two finite-semi-simple categories $\mathcal{C}, \mathcal{D}$ is the category $\mathcal{C} \boxtimes \mathcal{D}$ s.t.

$$\mathrm{Fun}(\mathcal{C} \boxtimes \mathcal{D}, \mathcal{E}) \simeq k\text{-Bilin}(\mathcal{C} \times \mathcal{D}, \mathcal{E})$$

All categories from now on ~~is~~ are finite-semisimple

fusion categories

$\mathcal{C}$ - monoidal is a fusion category if every object has left and right duals + $\mathbb{1}$ is simple.

examples G - finite group

1) Vec_G = G-graded vector spaces
2) $\mathrm{Rep}\, G$ = reps. of G

$\mathcal{C}$ - fusion. The Drinfeld centre of $\mathcal{C}$ is the braided fusion categor $Z_1(\mathcal{C})$:

Objects: $\{(X, c) \mid X \in \mathcal{C}\}$

$$c_Y : X \otimes Y \xrightarrow{\sim} Y \otimes X$$

1) $c_{\mathbb{1}} = \mathrm{id}$ 2) evident coherence diagram for triples.

<u>morphisms</u> : evident

<u>example</u> $\mathcal{C} = Vec_G$ = vector bundles on G

$Z_1(\mathcal{C})$ = Ad-invariant vector bundles on G.

<u>Module categories</u>

$\mathcal{C}$- fusion. A left <u>$\mathcal{C}$-module</u> is a category $\mathcal{M}$ equipped w/ a functor

$\triangleright : \mathcal{C} \boxtimes \mathcal{M} \longrightarrow \mathcal{M}$ satisfying obvious compatibilities.

Equivalently $\triangleright : \mathcal{C} \longrightarrow End(\mathcal{M})$ (monoidal)

A morphism of modules is defined in the obvious way. ~~A dir B~~ same for direct sums, indecomposable, etc..

<u>Program 1</u> classify indecomposables

Define bimodules in evident way

⇝ notion of Morita equivalent

<u>Thm</u> The center is an invariant under Morita equivalence.

⇝ Hochschild cohomology.