2d TFTs

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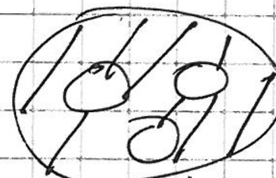
TFT ⇝ sym. monoidal functor $2\mathrm{Bord} \xrightarrow{Z} \mathcal{C}$

$\mathcal{C}$: objects : algebras in stable, presentable ∞-categories

1-morphisms : bimodule objects

2-morphisms: classifying spaces of bimodule maps.

- $Z(pt)$ will be self dual. ~~set $A = Z(pt)$~~
- $HH_*(Z(pt)) \simeq HH^*(Z(pt)) = Z(S^1)$
- Extra structure on $Z(S^1)$ coming from configurations of circles in circles



$$\rightsquigarrow Z(S^1)^{\otimes m} \longrightarrow Z(S^1)$$

$$Z \nearrow$$

$$(S^1)^{\sqcup m} \longrightarrow S^1$$

Let X be a perfect stack, over a field k, char $k = 0$.

$$Z(pt) = QCoh(X) \; ; \; Z(S^1) = QCoh(X) \otimes_{QCoh(X) \otimes QCoh(X)} QCoh(X)$$

Theorem (Ben-Zvi – Francis – Nadler)

Let $p_i : X_i \to Y$, $i = 1, 2$, be maps of perfect stacks. Then there are canonical equivs

$$QCoh(X_1 \times_Y X_2) \simeq QCoh(X_1) \otimes_{QCoh(Y)} QCoh(X_2)$$

$$\simeq Fun_{QCoh(Y)}(QCoh(X_1), QCoh(X_2))$$

Cor $QCoh(X)$ is self-dual as a $QCoh(Y)$-module (given $\pi : X \to Y$)

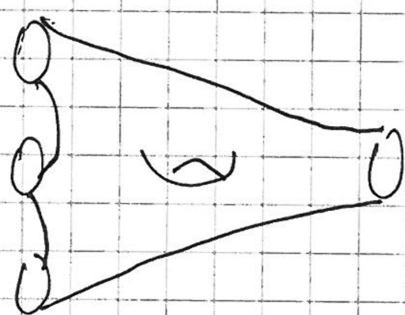
Apply theorem to

$$\mathrm{QCoh}(X) \otimes_{\mathrm{QCoh}(X) \otimes \mathrm{QCoh}(X)} \mathrm{QCoh}(X)$$

$$\downarrow \wr$$

$$\mathrm{QCoh}(\underbrace{X \times_{X \times X} X}_{\Omega X \,=\, LX \quad \text{loop space}})$$

$$Z(S^1) = \mathrm{QCoh}(LX)$$



$$(S^1)^{\sqcup 3} \longrightarrow S^1 \quad \rightsquigarrow \quad \mathrm{QCoh}(LX^3) \longrightarrow \mathrm{QCoh}(LX)$$

<u>why is this theorem true?</u>

start by looking at a product $X_1 \times X_2$

$$\boxtimes : \mathrm{QCoh}(X_1) \otimes \mathrm{QCoh}(X_2) \longrightarrow \mathrm{QCoh}(X_1 \times X_2)$$

$$\rightsquigarrow \quad \boxtimes : \mathrm{QCoh}(X_1)^c \otimes \mathrm{QCoh}(X_2)^c \longrightarrow \mathrm{QCoh}(X_1 \times X_2)^c$$

$\rightsquigarrow$ compact $\boxtimes$ compact generate $\mathrm{QCoh}(X_1 \times X_2)$

identify the two sides (of the Thm) as ~~modules~~ algebras for nice monads and construct an equivalence.