Math 748 Homework 2

Due Monday, September 18

1. In the polynomial ring $A = \mathbb{Q}[x, y]$, consider the principal ideal $\mathfrak{p} = (x^2 - y^3)$. Show that $\mathfrak{p}$ is a prime ideal (and thus $A/\mathfrak{p}$ is an integral domain), but $A/\mathfrak{p}$ is not integrally closed.

2. Let A be a subgring of a ring B, and let β be a unit in B. Show that every $\alpha \in A[\beta] \cap A[\beta^{-1}]$ is integral over A.

3. Is $\frac{3+2\sqrt{6}}{1-\sqrt{6}}$ an algebraic integer?

4. Determine the ring of integers of the field $\mathbb{Q}(\sqrt{d})$, where $d \in \mathbb{Z}$ is squarefree and $d \neq 0, 1$.

5. Let A be an integrally closed ring, and let K be its field of fractions. Let $f(x) \in A[x]$ be a monic polynomial. If $f(x)$ is reducible in $K[x]$, show that it is reducible in $A[x]$.